WORKSHEET FOR STUDENTS
RATIONALIZATION

1) Rationalize $\frac{5}{\sqrt{7}-\sqrt{2}}$

2) Rationalize $\frac{6}{\sqrt{72}}$

3) Rationalize $\frac{5}{\sqrt{125}}$

4) Rationalize $\frac{2h-4}{\sqrt{2+h}-2}$

5) Show that $\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}=1$

6) If $x = 4-\sqrt{15}$, find the value of $x+\frac{1}{x}$

7) Rationalize $\frac{\sqrt{x+7}-4}{x-9}$ by rationalizing the numerator.

8) Let $f(x)=\frac{\sqrt{x+4}-2}{x}$, $f(0)$ is not defined if we substitute $x = 0$ in $f(x)$ because $\frac{\sqrt{0+4}-2}{0}=\frac{0}{0}$ which is undefined.

Rationalize $f(x)$ by rationalizing the numerator, so that you have a real value for $f(0)$.

9) Rationalize $\frac{\sqrt{x+1}-1}{x}$ by rationalizing the numerator.

10) Let $f(x)=\frac{\sqrt{x+1}-2}{x-3}$. $f(3)$ is not defined because if we substitute $x = 3$ $f(x)$ *we get* $\frac{\sqrt{3+1}-2}{3-3}=\frac{0}{0}$ which is undefined.

Rationalize $f(x)$, *so* that you have a real value for $f(3)$.

11) Rationalize $\frac{4-\sqrt{x}}{16-x}$ by rationalizing the numerator.

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12) If $\frac{2+\sqrt{3}}{2-\sqrt{3}}=$ a+b$\sqrt{3}$ and a and b are rational numbers, find a and b.

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SOLUTIONS FOR STUDENTS

RATIONALIZATION

1) Rationalize $\frac{5}{\sqrt{7}-\sqrt{2}}$

Solution: The conjugate of $\sqrt{7}-\sqrt{2}$ is $\sqrt{7}+\sqrt{2}$. So, we multiply both the numerator and denominator of $\frac{5}{\sqrt{7}-\sqrt{2}}$ by $\sqrt{7}+\sqrt{2}$.

$$\frac{5}{\sqrt{7}-\sqrt{2}} \cdot \frac{(\sqrt{7}+\sqrt{2})}{(\sqrt{7}+\sqrt{2})} = \frac{5(\sqrt{7}+\sqrt{2})}{(\sqrt{7})^2-(\sqrt{2})^2} = \frac{5(\sqrt{7}+\sqrt{2})}{7-2} = \frac{5(\sqrt{7}+\sqrt{2})}{5} = \sqrt{7}+\sqrt{2}$$

2) Rationalize $\frac{6}{\sqrt{72}}$

Solution: Here $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

So, $\frac{6}{\sqrt{72}} = \frac{6}{6\sqrt{2}}$

We multiply both denominator and Numerator by $\sqrt{2}$.

$$\frac{6}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{6(2)} = \frac{6\sqrt{2}}{12} = \frac{\sqrt{2}}{2}$$

3) Rationalize $\frac{5}{\sqrt{125}}$.

Solution: $\sqrt{125} = \sqrt{25.5} = 5\sqrt{5}$

So, $\frac{5}{\sqrt{125}} = \frac{5}{5\sqrt{5}} = \frac{1}{5\sqrt{5}}$

To rationalize, multiply the numerator and Denominator by $\sqrt{5}$.

So, $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

4) Rationalize $\frac{2h-4}{\sqrt{2+h}-2}$

Solution: The Conjugate of $\sqrt{2+h}-2$ is $\sqrt{2+h}+2$

So, we multiply the numerator and denominator by $\sqrt{2+\mathrm{h}}+2$

$$\frac{2h-4}{\sqrt{2+h}-2}\cdot\frac{\sqrt{2+h}+2}{\sqrt{2+h}+2}=\frac{2(h-2)(\sqrt{2+h}+2)}{(\sqrt{2+h})^2-(2)^2}$$

$$=\frac{2(h-2)(\sqrt{2+h}+2)}{2+h-4}=\frac{2(h-2)(\sqrt{2+h}+2)}{h-2}=2(\sqrt{2+h}+2)$$

5)Show that $\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}=1$

Solution: Let us consider the expression one by one.

a) $\frac{1}{3+\sqrt{7}}=\frac{1}{3+\sqrt{7}}\frac{(3-\sqrt{7})}{(3-\sqrt{7})}=\frac{3-\sqrt{7}}{(3)^2-(\sqrt{7})^2}=\frac{3-\sqrt{7}}{9-7}=\frac{3-\sqrt{7}}{2}$

Here we multiplied the numerator and denominator by the conjugate of $3+\sqrt{7}$ which is $3-\sqrt{7}$

b) $\frac{1}{\sqrt{7}+\sqrt{5}}=\frac{1}{\sqrt{7}+\sqrt{5}}\frac{(\sqrt{7}-\sqrt{5})}{(\sqrt{7}-\sqrt{5})}=\frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7})^2-(\sqrt{5})^2}=\frac{\sqrt{7}-\sqrt{5}}{7-5}=\frac{\sqrt{7}-\sqrt{5}}{2}$

Here we multiplied the numerator and denominator by the conjugate of $\sqrt{7}+\sqrt{5}$ which is $\sqrt{7}-\sqrt{5}$.

c) $\frac{1}{\sqrt{5}+\sqrt{3}}=\frac{1}{\sqrt{5}+\sqrt{3}}\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}=\frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}=\frac{\sqrt{5}-\sqrt{3}}{5-3}=\frac{\sqrt{5}-\sqrt{3}}{2}$

Here we multiplied the numerator and denominator by the conjugate of $\sqrt{5}+\sqrt{3}$ which is $\sqrt{5}-\sqrt{3}$

d) $\frac{1}{\sqrt{3}+1}\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}=\frac{\sqrt{3}-1}{(\sqrt{3})^2-(1)^2}=\frac{\sqrt{3}-1}{3-1}=\frac{\sqrt{3}-1}{2}$

Here we multiplied the numerator and denominator by the conjugate of $\sqrt{3}+1$ which is $\sqrt{3}-1$

So, $\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}=\frac{3-\sqrt{7}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+\frac{\sqrt{5}-\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}$

$=\frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}=\frac{2}{2}=1$

6) If $x=4-\sqrt{15}$ find the value of $x+\frac{1}{x}$.

Solution: $\frac{1}{x}=\frac{1}{4-\sqrt{15}}$

The conjugate of $4-\sqrt{15}$ *is* $4+\sqrt{15}$. So, we multiply the numerator and denominator by $4+\sqrt{15}$.

$$\frac{1}{x} = \frac{1}{4-\sqrt{15}} \cdot \frac{4+\sqrt{15}}{4+\sqrt{15}} = \frac{4+\sqrt{15}}{(4)^2-(\sqrt{15})^2} = \frac{4+\sqrt{15}}{16-15} = 4+\sqrt{15}$$

So, $x + \frac{1}{x} = 4 - \sqrt{15} + 4 + \sqrt{15} = 8.$

7) Rationalize $\frac{\sqrt{x+7}-4}{x-9}$ by rationalizing the numerator.

Solution: The conjugate of $\sqrt{x+7}$ - 4 is $\sqrt{x+7}+4$

So, we multiply the numerator and denominator by $\sqrt{x+7}+4$

$$\frac{\sqrt{x+7}-4}{x-9} \cdot \frac{(\sqrt{x+7}+4)}{(\sqrt{x+7}+4)} = \frac{(\sqrt{x+7})^2 - (4)^2}{(x-9)(\sqrt{x+7}+4)} = \frac{x+7-16}{(x-9)(\sqrt{x+7}+4)}$$

$$= \frac{x-9}{(x-9)(\sqrt{x+7}+4)} = \frac{1}{\sqrt{x+7}+4}$$

8) Let $f(x) = \frac{\sqrt{x+4}-2}{x}$, $f(0)$ is not defined if we substitute $x = 0$ in $f(x)$because $\frac{\sqrt{0+4}-2}{0} = \frac{0}{0}$ which is undefined.

Rationalize $f(x)$ by rationalizing the numerator, so that you have a real value for $f(0)$.

Solution: The conjugate of $\sqrt{x+4}$ -2 is $\sqrt{x+4}+2$

So, we multiply the numerator and denominator by $\sqrt{x+4}+2$

$$\frac{\sqrt{x+4}-2}{x} \cdot \frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)} = \frac{(\sqrt{x+4})^2 - (2)^2}{x(\sqrt{x+4}+2)} = \frac{x+4-4}{x\sqrt{x+4}+2)}$$

$$= \frac{x}{x\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

9) Rationalize $\frac{\sqrt{x+1}-1}{x}$ by rationalizing the numerator.

Solution: The conjugate of $\sqrt{x+1}$ -1 is $\sqrt{x+1}+1$

So, we multiply the numerator and denominator by $\sqrt{x+1}+1$

$$\frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \frac{(\sqrt{x+1})^2 - (1)^2}{x(\sqrt{x+1}+1)} = \frac{x+1-1}{x\sqrt{x+1}+1)}$$

$$= \frac{x}{x\sqrt{x+1}+1)} = \frac{1}{\sqrt{x+1}+1}$$

So, $f(x)$ after rationalization equals $\frac{1}{\sqrt{x+1}+1}$

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$f(0) = \frac{1}{\sqrt{0+1}+1} = \frac{1}{1+1} = \frac{1}{2}$

10) Let $f(x) = \frac{\sqrt{x+1}-2}{x-3}$. $f(3)$ is not defined because if we substitute $x = 3$ $f(x)we\ get$ $\frac{\sqrt{3+1}-2}{3-3} = \frac{0}{0}$ which is undefined.

Rationalize $f(x)$, *so* that you have a real value for $f(3)$.

Solution: The conjugate of $\sqrt{x+1}$ - 2 is $\sqrt{x+1}$+2

So, we multiply the numerator and denominator by $\sqrt{x+1}$+2

$$\frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{\left(\sqrt{x+1}\right)^2-(2)^2}{(x-3)(\sqrt{x+1}+2)} = \frac{x+1-4}{(x-3)\sqrt{x+1}+2)}$$

$$= \frac{x-3}{(x-3)\sqrt{x+1}+2)} = \frac{x-3}{(x-3)\sqrt{x+1}+2} = \frac{1}{\sqrt{x+1}+2}$$

So, f(x) after rationalization equals $\frac{1}{\sqrt{x+1}+2}$

Now $f(3) = \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \frac{1}{4}$

11) Rationalize $\frac{4-\sqrt{x}}{16-x}$ by rationalizing the numerator.

Solution: The conjugate of $4 - \sqrt{x}\ is\ 4 + \sqrt{x}$

we multiply the numerator and denominator by $4 + \sqrt{x}$

$$\frac{4-\sqrt{x}}{16-x} \frac{(4-\sqrt{x})}{(4+\sqrt{x})} = \frac{(4)^2 - (\sqrt{x})^2}{(16-x)(4+\sqrt{x})} = \frac{16-x}{(16-x)(4+\sqrt{x})} = \frac{1}{4+5x}$$

12) If $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ = a+b$\sqrt{3}$ and a and b are rational numbers, find a and b.

Solution: The conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$

we multiply the numerator and denominator by $2 + \sqrt{3}$

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2)^2 - (\sqrt{3})^2} = \frac{4+2\sqrt{3}+2\sqrt{3}+3)}{4-3} = \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$$

So, a=7 and b=4

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