RATIONAL EQUATIONS
WORKSHEET FOR STUDENTS

Solve each Radical equation and check for extraneous solutions, if any.

1. $x + 2 = \frac{15}{x}$
2. $x + \frac{4x}{x-3} = \frac{12}{x-3}$
3. $x + \frac{10}{x} = 7$
4. $x+2=\frac{15}{x}$
5. $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$
6. $\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x^2-x}$
7. $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$
8. $\frac{2n^2 - 8n - 10}{5n} - 1 = \frac{n+6}{5n}$
9. $\frac{\vartheta - 6}{2\vartheta + 2\vartheta - 4} + \frac{\vartheta}{2\vartheta - 2} = \frac{1}{2}$
10. $\frac{1}{x^2-5x} = \frac{x+7}{x} - 1$

RATIONAL EQUATIONS
SOLUTION FOR STUDENTS

1) $x + 2 = \frac{15}{x}$

⇨ $\frac{x}{1} + \frac{2}{1} = \frac{15}{x}$ **(1)**

LCD $(1,1,x) = x$

We multiply (1)by x

$\left(\frac{x}{1} + \frac{2}{1} = \frac{15}{x}\right)x$ ⇨ $x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$ ⇨ $(x - 3)(x + 5) = 0$ ⇨ $x = 3$ or $x = -5$

Check for x=3

Substitute $x = 3$ in the equation $x + 2 = \frac{15}{x}$

$3+2 = \frac{15}{3}$ ⇨ $5=5$

So, $x = 3$ is a Solution.

Check for x= -5

Substitute $x = -5$ in the equation $x + 2 = \frac{15}{x}$

$-5+2 = \frac{15}{-5}$ ⇨ $-3= -3$

So, $x = -3$ is a Solution.

2) $x + \frac{4x}{x-3} = \frac{12}{x-3}$

⇨ $\frac{x}{1} + \frac{4x}{x-3} = \frac{12}{x-3}$ **(1)**

LCD$(1, x - 3, x - 3) = x - 3$

Multiply (1) by the (x -3)

$\left(\frac{x}{1} + \frac{4x}{x-3} = \frac{12}{x-3}\right)$x-3

$x(x - 3) + 4x = 12$ ⇨ $x^2 + x - 12 = 0$

$= (x + 4)(x - 3) = 0$ ⇨ $x = -4$ or $x = 3$

Check for x= -4

Substitute $x = -4$ in the equation $x + \frac{4x}{x-3} = \frac{12}{x-3}$

$-4+\frac{4(-4)}{-4-3} = \frac{12}{-4-3}$ ⇨ $-4+\frac{16}{7} = \frac{12}{-7}$ ⇨ $\frac{12}{-7}$ ⇨ $\frac{12}{-7}$

So, $x = -4$ is a solution.

Check for x= 3

Substitute $x = 3$ in the equation $x + \frac{4x}{x-3} = \frac{12}{x-3}$

We get a zero denominator because of the presence of

$$\frac{4x}{x-3} \text{ and } \frac{12}{x-3}$$

So, $x = 3$ is an extraneous solution.

3) $x + \frac{10}{x} = 7$

$\frac{x}{1} + \frac{10}{x} = \frac{7}{1}$ (1)

LCD $(1, x, 1) = x$

Multiply (1) by the x

$\left(\frac{x}{1} + \frac{10}{x} = \frac{7}{1}\right)x$ ⇨ $x^2 + 10 = 7x$

⇨ $x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$ ⇨ $x = 5$ or $x = 2$

Check for x= 5

Substitute $x = 5$ in the equation $x + \frac{10}{x} = 7$

$5+\frac{10}{5} = 7$ ⇨ $7 = 7$

So, $x = 5$ is a solution.

Check for x= 2

Substitute $x = 2$ in $x + \frac{10}{x} = 7$

$2 + \frac{10}{2} = 7$ ⇨ $7 = 7$

So, $x = 2$ is a solution.

4) $x+2=\frac{15}{x}$

$\frac{x}{1}+\frac{2}{1}=\frac{15}{x}$ (1)

LCD $(1,1,x) = x$

Multiply (1) by the x

$$\left(\frac{x}{1}+ \frac{2}{1} = \frac{15}{x}\right)x$$

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

So, x =-5 0r x =3

Check for x= -5

Substitute $x = -5$ in the equation $x + 2 = \frac{15}{x}$

$-5+2=\frac{15}{-5}$ ⇨ $-3 = -3$

So, $x = -5$ is a solution.

Check for x= 3

Substitute $x = 3$ in the equation $x + 2 = \frac{15}{x}$

$3+2=\frac{15}{3}$ ⇨ $5 = 5$

So, $x = 3$ is a solution.

5. $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

After factoring we get $\frac{2}{1} - \frac{3}{x+4} = \frac{12}{x(x+4)}$ (1)

LCD [1, x+4, x(x+4)]=x(x+4)

Multiply (1) by the $x(x + 4)$

$$\left(\frac{2}{1} - \frac{3}{x+4} = \frac{12}{x(x+4)}\right)x(x + 4)$$

$2x(x + 4) - 3x = 12$ ⇨ $2x^2 + 8x - 3x - 12 = 0$

⇨ $2x^2 + 5x - 12 = 0$ ⇨ $2x^2 + 5x - 12 = 0$

$(2x - 3)(x + 4) = 0$ ⇨ $x = \frac{3}{2}$ or $x = -4$

Check for $x = \frac{3}{2}$

Substitute $x = \frac{3}{2}$ in the equation $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

$2 - \frac{3}{\frac{3}{2}+4} = \frac{12}{\left(\frac{3}{2}\right)^2+4\left(\frac{3}{2}\right)} \Rightarrow 2 - \frac{3}{\frac{11}{2}} = \frac{12}{\frac{9}{4}+6}$

$2 - \frac{6}{11} = \frac{12}{\frac{33}{4}} \quad \Rightarrow \quad \frac{16}{11} = \frac{16}{11}$

So, $x = \frac{3}{2}$ is a Solution

<u>Check for x= -4</u>

Substitute $x = -4$ *in* $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

We get a zero denominator because of the presence of $\frac{3}{x+4}$

So, $x = -4$ is an extraneous Solution.

6) $\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x^2-x}$

After factoring we get $\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x(x-1)}$ (**1**)

LCD [x, x-1, x(x-1)] = x(x-1)

Multiply (1) by the $x(x-1)$

$\left(\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x(x-1)}\right) x(x-1)$

$(x+2)(x-1) - 4x = -2$

$\Rightarrow x^2 + x - 2 - 4x + 2 = 0$

$\Rightarrow x^2 - 3x = 0$

$\Rightarrow x = 0 \quad$ or $x = 3$

<u>Check for x= 0</u>

Substitute $x = 0$ in $\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x(x-1)}$

Here we get a zero denominator because of the presence of $\frac{x+2}{x}$

So, $x = 0$ is an extraneous solution.

<u>Check for x= 3</u>

Substitute $x = 3$ in $\frac{x+2}{x} - \frac{4}{x-1} = \frac{-2}{x(x-1)}$

$\frac{3+2}{3} - \frac{4}{3-1} = \frac{-2}{3^2-3} \Rightarrow \frac{5}{3} - \frac{4}{2} = \frac{-2}{6}$

$\Rightarrow \frac{-1}{3} = \frac{-1}{3}$
So, $x = 3$ is a Solution.

7) $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

After factoring we get $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x(x+3)}$ (1)

LCD [x, x+31, x(x+3)] = x(x+3)

Multiply (1) by x(x+3)

$\left(\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x(x+3)}\right) x(x+3)$

$(x+3)^2 - 2x = 6 \Rightarrow x^2 + 6x + 9 - 2x = 6$

$\Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+1)(x+3) = 0$

$\Rightarrow x = -1$ or $x = -3$

Check for x= -1

Substitute $x = -1$ in $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

$\frac{-1+3}{-1} - \frac{2}{-1+3} = \frac{6}{(-1)^2+3(-1)}$

$\Rightarrow$ -2-1= $\frac{6}{-2} \Rightarrow$ -3 =-3

So, x = -1 is a solution.

Check for x= -3

Substitute $x = -3$ in $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

Here we get a zero denominator because of the presence of $\frac{x+3}{x}$

So, x = -3 is an extraneous solution.

8) $\frac{2n^2 - 8n - 10}{5n} - 1 = \frac{n+6}{5n}$

$\frac{2n^2 - 8n - 10}{5n} - \frac{1}{1} = \frac{n+6}{5n}$ **(1)**

LCD (5n, 1, 5n) = 5n

Multiply (1) by 5n

$$\left(\frac{2n^2 - 8n - 10}{5n} - \frac{1}{1} = \frac{n+6}{5n}\right)5n$$

$2n^2 - 8n - 10 - 5n = n + 6$
$2n^2 - 13n - 10 = n + 6$
$2n^2 - 14n - 16 = 0 \Rightarrow n^2 - 7n - 8 = 0$
$(n - 8)(n + 1) = 0$
$n = 8$ or $n = -1$

Check for n=8

Substitute $n = 8$ in the equation $\frac{2n^2 - 8n - 10}{5n} - 1 = \frac{n+6}{5n}$

$\frac{2(8)^2 - 8(8) - 10}{5(8)} - 1 = \frac{8+6}{5(8)} \Rightarrow \frac{128-64-10}{40} - 1 = \frac{14}{40}$

$\frac{54}{40} - 1 = \frac{14}{40}$
$\Rightarrow \frac{14}{40} = \frac{14}{40}$
$\Rightarrow \frac{7}{20} = \frac{7}{20}$
So, $x = 8$ is a Solution

Check for n=-1

Substitute $n = -1$ in the equation $\frac{2n^2 - 8n - 10}{5n} - 1 = \frac{n+6}{5n}$

$\frac{2(-1)^2 - 8(-1) - 10}{5(-1)} - 1 = \frac{-1+6}{5(-1)}$
$\Rightarrow \frac{2+8-10}{-5} - 1 = \frac{5}{-5}$
$\Rightarrow 0 - 1 = -1$
$\Rightarrow -1 = -1$
So, $n = -1$ is a solution.

9) $\frac{\vartheta - 6}{2\vartheta^2 + 2\vartheta - 4} + \frac{\vartheta}{2\vartheta - 2} = \frac{1}{2}$

After factoring we get

$$\frac{\vartheta - 6}{2(\vartheta + 2)(\vartheta - 1)} + \frac{\vartheta}{2(\vartheta - 1)} = \frac{1}{2} \qquad (1)$$

LCD $[\ 2(\vartheta + 2)(\vartheta - 1), 2(\vartheta - 1), 2] = 2(\vartheta - 1)(\vartheta + 2)$

Multiply (1) by $2(\vartheta - 1)(\vartheta + 2)$.

$$\left(\frac{\vartheta - 6}{2(\vartheta + 2)(\vartheta - 1)} + \frac{\vartheta}{2(\vartheta - 1)} = \frac{1}{2}\right)2(\vartheta - 1)(\vartheta + 2)$$

$\vartheta - 6 + \vartheta(\vartheta + 2) = (\vartheta - 1)(\vartheta + 2)$

$\vartheta - 6 + \vartheta^2 + 2\vartheta = \vartheta^2 + \vartheta - 2$

$3\,\vartheta - 6 = \vartheta - 2 \Rightarrow 2\vartheta = 4 \Rightarrow \vartheta = 2$

Check for $\vartheta = 2$

Substitute $\vartheta = 2$ in the equation $\frac{\vartheta - 6}{2\vartheta^2 + 2\vartheta - 4} + \frac{\vartheta}{2\vartheta - 2} = \frac{1}{2}$

$\frac{2-6}{2(2)^2+2(2)-4} + \frac{2}{2(2)-2} = \frac{1}{2}$

$\Rightarrow \frac{-4}{8+4-4} + \frac{2}{2} \Rightarrow \frac{1}{2}$

$\frac{-4}{8} + 1 = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$

So, $\vartheta = 2$ is a Solution

10) $\frac{1}{x^2 - 5x} = \frac{x+7}{x} - 1$

After factoring we get $\frac{1}{x(x-5)} = \frac{x+7}{x} - \frac{1}{1}$ **(1)**

LCD (x(x-5), x, 1) = x(x-5)

Multiply (1) by the $x(x - 5)$ we get

$\left(\frac{1}{x(x-5)} = \frac{x+7}{x} - 1\right) x(x - 5)$

$1 = (x + 7)(x - 5) - x(x - 5)$

$1 = x^2 + 2x - 35 - x^2 + 5x$

$1 = 7x - 35$

$\Rightarrow x = \frac{36}{7}$

Check for $\mathbf{x = \frac{36}{7}}$

We substitute $x = \frac{36}{7}$ in $\frac{1}{x^2 - 5x} = \frac{x+7}{x}$ -1

$$\frac{1}{\left(\frac{36}{7}\right)^2 - 5\left(\frac{36}{7}\right)} = \frac{\left(\frac{36}{7}\right)+7}{\left(\frac{36}{7}\right)} - 1$$

⇨ $\frac{1}{\frac{1296}{49} - \frac{180}{7}} = \frac{\frac{85}{7}}{\frac{36}{7}}$ -1

⇨ $\frac{1}{\frac{36}{49}} = \frac{85}{36}$ - 1

⇨ $\frac{49}{36} = \frac{49}{36}$

So, $x = \frac{36}{7}$ is a Solution