RATIONALIZATION
WORKSHEET FOR INSTRUCTOR

1) Rationalize $\frac{6}{\sqrt{11}-\sqrt{5}}$

2) Rationalize $\frac{11}{\sqrt{242}}$

3) Rationalize $\frac{h}{\sqrt{3+h}-\sqrt{3}}$

4) Rationalize $\frac{17}{6-\sqrt{2}}$

5) Show that $\frac{1}{\sqrt{5}-\sqrt{7}}+\frac{1}{\sqrt{3}-\sqrt{7}}=\frac{3-\sqrt{5}}{2}$

6) If $x=6-\sqrt{35}, find\ the\ value\ of\ x+\frac{1}{x}$

7) Rationalize $\frac{3x-15}{\sqrt{x+11}-4}$

8) Let $f(x)=\frac{\sqrt{x+64}-8}{x}$, $f(0)$ is not defined because

if we substitute substitute x $=0$ in $f(x)$ we get $\frac{\sqrt{0+64}-8}{0}=\frac{0}{0}$

which is undefined.

9) Find a and b if $\frac{2+6\sqrt{5}}{2-6\sqrt{6}}=a+b\sqrt{5}$.

10) Let $f(x)=\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$. $f(2)$ is not defined because

when you substitute $x=0$ we get $\frac{\sqrt{5}-\sqrt{7}}{0}$ which is

undefined. Rationalize $f(x)$ by rationalizing the numerator

and get value of $f(2)$.

11) Let $f(x) = \frac{4x^2 - 64}{2\sqrt{x} - 4}$. $f(4)$ is not defined because when you substitute $x = 0$ we get $\frac{0}{0}$ which is undefined.

Rationalize $f(x)$ by rationalizing the numerator and get value of $f(4)$.

SOLUTIONS FOR INSTRUCTORS
RATIONALIZATON

1)Rationalize $\frac{6}{\sqrt{11}-\sqrt{5}}$

Solution: The conjugate of $\sqrt{11}-\sqrt{5}$ *is* $\sqrt{11}+\sqrt{5}$. We multiply the numerator and denominator of $\frac{6}{\sqrt{11}-\sqrt{5}}$ by $\sqrt{11}+\sqrt{5}$.

$$\frac{6}{\sqrt{11}-\sqrt{5}}\cdot\frac{(\sqrt{11}+\sqrt{5})}{(\sqrt{11}+\sqrt{5})}=\frac{6(\sqrt{11}+\sqrt{5})}{(\sqrt{11})^2-(\sqrt{5})^2}=\frac{6(\sqrt{11}+\sqrt{5})}{11-5}$$

$$=\frac{6(\sqrt{11}+\sqrt{5})}{6}=\sqrt{11}+\sqrt{5}$$

2)Rationalize $\frac{11}{\sqrt{242}}$

Solution: Here $\sqrt{242}=\sqrt{121\times 2}=11\sqrt{2}$

So, $\frac{11}{\sqrt{242}}=\frac{11}{11\sqrt{2}}=\frac{1}{\sqrt{2}}$

We rationalize the denominator by Multiplying $\frac{1}{\sqrt{2}}$ by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

3)Rationalize $\frac{h}{\sqrt{3+h}-\sqrt{3}}$

Solution: We rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{3+h}+\sqrt{3}$

$$\frac{h}{\sqrt{3+h}-\sqrt{3}}\cdot\frac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}}=\frac{h(\sqrt{3+h}+\sqrt{3})}{(\sqrt{3+h})^2-(\sqrt{3})^2}$$

$$=\frac{h(\sqrt{3+h}+\sqrt{3})}{3+h-3}=\frac{h(\sqrt{3+h}+\sqrt{3})}{h}=\sqrt{3+h}+\sqrt{3}$$

4)Rationalize $\frac{17}{6-\sqrt{2}}$

Solution: We rationalize the denominator by multiplying both the numerator and denominator by the conjugate of $6-\sqrt{2}$ which is $6+\sqrt{2}$

$$\frac{17}{6-\sqrt{2}} \cdot \frac{6+\sqrt{2}}{6+\sqrt{2}} = \frac{17(6+\sqrt{2})}{(6)^2-(\sqrt{2})^2} = \frac{17(6+\sqrt{2})}{36-2} = \frac{17(6+\sqrt{2})}{34} = \frac{6+\sqrt{2}}{2}$$

5)Show that $\frac{1}{\sqrt{5}-\sqrt{7}} + \frac{1}{3-\sqrt{7}} = \frac{3+\sqrt{5}}{2}$

Solution: First, we Rationalize $\frac{1}{\sqrt{5}-\sqrt{7}}$ by multiplying the numerator and denominator by the conjugate of $\sqrt{5}-\sqrt{7}$ which is $\sqrt{5}+\sqrt{7}$

$$\frac{1}{\sqrt{5}-\sqrt{7}} \cdot \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} = \frac{\sqrt{5}+\sqrt{7}}{(\sqrt{5})^2-(\sqrt{7})^2} = \frac{\sqrt{5}+\sqrt{7}}{5-7} = \frac{\sqrt{5}+\sqrt{7}}{-2}$$

Similarly, we rationalize $\frac{1}{3-\sqrt{7}}$ by multiplying the numerator and denominator by the conjugate of $3-\sqrt{7}$ which is $3+\sqrt{7}$.

$$\frac{1}{3-\sqrt{7}} = \frac{1}{3-\sqrt{7}} \cdot \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3+\sqrt{7}}{3^2-(\sqrt{7})^2} = \frac{3+\sqrt{7}}{9-7} = \frac{3+\sqrt{7}}{2}$$

So, $\frac{1}{\sqrt{5}-\sqrt{7}} + \frac{1}{\sqrt{3}-\sqrt{7}} = \frac{\sqrt{5}+\sqrt{7}}{-2} + \frac{3+\sqrt{7}}{2} = \frac{-\sqrt{5}-\sqrt{7}+3+\sqrt{7}}{2} = \frac{3-\sqrt{5}}{2}$

6) If $x = 6-\sqrt{35}$, find the value of $x+\frac{1}{x}$.

Solution: Here, $\frac{1}{x} = \frac{1}{6-\sqrt{35}}$

We multiply the numerator and denominator of $\frac{1}{6-\sqrt{35}}$ by the conjugate of $6-\sqrt{35}$ which is $6+\sqrt{35}$.

$$\frac{1}{6-\sqrt{35}} = \frac{1}{6-\sqrt{35}} \cdot \frac{6+\sqrt{35}}{6+\sqrt{35}} = \frac{6+\sqrt{35}}{(6)^2-(\sqrt{35})^2} = \frac{6+\sqrt{35}}{36-35} = 6+\sqrt{35}$$

So, $x+\frac{1}{x} = 6-\sqrt{35}+6+\sqrt{35} = 12$.

7) Rationalize $\frac{3x-15}{\sqrt{x+11}-4}$

Solution: The conjugate of $\sqrt{x+11}$ - 4 is $\sqrt{x+11}+4$

So, we multiply the numerator and denominator by $\sqrt{x+11}+4$

$$\frac{3x-15}{\sqrt{x+11}-4}=\frac{3(x-5)}{\sqrt{x+11}-4}\cdot\frac{\sqrt{x+11}+4}{\sqrt{x+11}+4}$$

$$=\frac{3(x-5)(\sqrt{x+11}+4)}{(\sqrt{x+11})^2-(4)^2}=\frac{3(x-5)(\sqrt{x+11}+4)}{x+11-16}$$

$$=\frac{3(x-5)(\sqrt{x+11}+4)}{x-5}=3(\sqrt{x+11}+4)$$

8) Rationalize $f(x)=\frac{\sqrt{x+64}-8}{x}$

Solution: We Rationalize the numerator by multiplying the Numerator and denominator by the Conjugate of $\sqrt{x+64}$ - 8 which is $\sqrt{x+64}$ +8

$$\frac{\sqrt{x+64}-8}{x}\cdot\frac{\sqrt{x+64}+8}{\sqrt{x+64}+8}=\frac{(\sqrt{x+64})^2-(8)^2}{x(\sqrt{x+64}+8)}=\frac{x+64-64}{x\sqrt{x+64}+8)}$$

$$=\frac{x}{x(\sqrt{x+64}+8)}=\frac{1}{\sqrt{x+64}+8}$$

So, now f $(x)=\frac{1}{\sqrt{x+64}+8}$

So, now f 0) $=\frac{1}{\sqrt{0+64}+8}=\frac{1}{8+8}=\frac{1}{16}$

9) Find a and b if $\frac{2+6\sqrt{5}}{2-6\sqrt{5}}=a+b\sqrt{5}$.

Solution: We Rationalize the numerator by multiplying the Numerator and denominator by the Conjugate of 2- $6\sqrt{5}$ which is $2+6\sqrt{5}$.

$$\frac{2+6\sqrt{5}}{2-6\sqrt{5}}\cdot\frac{(2+6\sqrt{5})}{(2+6\sqrt{5})}=\frac{4+12\sqrt{5}+12\sqrt{5}+180}{(6)^2-(6\sqrt{5})^2}=\frac{184+24\sqrt{5}}{(6)^2-(6\sqrt{5})^2}=\frac{184+24\sqrt{5}}{36-180}$$

$$=\frac{184+24\sqrt{5}}{-144}=\frac{4(46+6\sqrt{5})}{-144}=\frac{46+6\sqrt{5}}{-36}=\frac{46}{-36}+\frac{6\sqrt{5}}{-36}=\frac{-23}{18}-\frac{\sqrt{5}}{6}.$$

So, a $= \frac{-23}{18}$ and b $= \frac{-1}{6}$

10. $f(x) = \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$

Rationalize $f(x)$ by multiplying the numerator and denominator by $\sqrt{2x+5} + \sqrt{x+7}$.

. $f(x) = \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \frac{(\sqrt{2x+5}+\sqrt{x+7})}{(\sqrt{2x+5}+\sqrt{x+7})} = \frac{(\sqrt{2x+5})^2 - (\sqrt{x+7})^2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$

$= \frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})} = \frac{x-2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})} = \frac{1}{\sqrt{2x+5}+\sqrt{x+7}}$

So, f(x) $= \frac{1}{\sqrt{2x+5}+\sqrt{x+7}}$

$\Rightarrow f(2) = \frac{1}{\sqrt{2(2)+5}+\sqrt{2+7}} = \frac{1}{6}$.

11) Let $f(x) = \frac{4x^2-64}{2\sqrt{x}-4}$. $f(4)$ is not defined because when you substitute $x = 0$ we get $\frac{0}{0}$ which is undefined.

Rationalize $f(x)$ by rationalizing the numerator and get value of $f(4)$.

Solution: $f(x) = \frac{4x^2-64}{2\sqrt{x}-4} = \frac{4(x^2-16)}{2(\sqrt{x}-2)} = \frac{2(x-4)(x+4)}{\sqrt{x}-2}$

We rationalize f(x) by multiplying numerator and denominator $\sqrt{x} - 2$.

$\frac{2(x-4)(x+4)}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{2(x-4)(x+4)(\sqrt{x}+2)}{(\sqrt{x})^2-(2)^2} = \frac{2(x-4)(x+4)(\sqrt{x}+2)}{x-4}$

$= 2(x+4)(\sqrt{x} + 2)$

So, $f(x) = 2(x+4)(\sqrt{x} + 2)$

So, $f(4) = 2(4 + 4)(\sqrt{4} + 2) = 64$.